Georgescu-Roegen, Nicholas (1971) *The Entropy Law and the Economic Process.* Cambridge: Harvard University Press.

CHAPTER II *Science, Arithmomorphism, and Dialectics*

1. "No Science without Theory." Theoretical science having the marvelous qualities just described, we can easily understand the sanguine hopes raised by Newton's success in transforming mechanics into such a science. At last, some two thousand years after Euclid's *Elements*, Newton's *Principia Mathematica* proved that theoretical science can grow in other domains besides geometry, and equally well. But sanguine hopes are sanguine hopes: thoughts on the matter, especially of those fascinated most by the powers of Logic, became prey to the confusion between "some fields" and "all fields." In the end almost everybody interpreted the evidence as proof that knowledge in *all* fields can be cast into a theoretical mold. Especially after the astounding discovery of Neptune "at the tip of Leverrier's pen," spirits ran high in all disciplines, and one scientist after another announced his intention of becoming the Newton of his own science. François Magendie aspired to place even physiology "on the same sure footing" as mechanics.[1] "Thus the confusion of tongues"—as one economist lamented—"was propagated from science to science."[2]

On the whole, the scientific temper has not changed much. To be sure, the position that mechanics constitutes the only road leading to divine knowledge—as Laplace argued in his magnificent apotheosis[3]—has been

[1] J. M. D. Olmsted and E. H. Olmsted, *Claude Bernard and the Experimental Method in Medicine* (New York, 1952), p. 23.

[2] S. Bauer, quoted in J. S. Gambs, *Beyond Supply and Demand* (New York, 1946), p. 29n. My translation.

[3] P. S. Laplace, *A Philosophical Essay on Probabilities* (New York, 1902), p. 4.

officially abandoned by almost every special science. Curiously, the move was not caused by the recognition of the failures following the adoption of this position outside physics, but induced by the fact that physics itself had to reject it.[4] In place of "all sciences must imitate mechanics," the battle cry of the scholarly army is now "no science without theory." But the change is rather skin deep, for by "theory" they usually mean a logical file of knowledge as exemplified only by geometry and mechanics.[5]

No other science illustrates better than economics the impact of the enthusiasm for mechanistic epistemology upon its evolution. Does the transforming of economics into "a physico-mathematical science" require a measure of utility which escapes us? "*Eh bien!*"—exclaimed Walras characteristically—"this difficulty is not insurmountable. Let us suppose that this measure exists, and we shall be able to give an exact and mathematical account" of the influence of utility on prices, etc.[6] Unfortunately, this uncritical attitude has ever since constituted the distinct flavor of mathematical economics. In view of the fact that theoretical science is a living organism, it would not be exaggerating to say that this attitude is tantamount to planning a fish hatchery in a moist flower bed.

Jevons showed some concern over whether the new environment—the economic field—would contain the basic elements necessary for the theoretical organism to grow and survive. Indeed, before declaring his intention to rebuild economics as "*the mechanics of utility and self-interest*," he took pains to point out that in the domain of economic phenomena there is plenty of quantitative "moisture" in "the private-account books, the great ledgers of merchants and bankers and public offices, the share lists, price lists, bank returns, monetary intelligence, Custom-house and other Government returns."[7] But Jevons, like many others after him, failed to go on to explain how ordinary statistical data could be substituted for the variables of his mechanical equations. By merely expressing the hope that statistics might become "more complete and accurate . . . so that the formulae could be endowed with exact meaning,"[8] Jevons set an often-followed pattern for avoiding the issue.

Certainly, after it was discovered that theoretical science can function properly in another domain besides geometry, scientists would have been

[4] See chapter "The Decline of the Mechanical View" in A. Einstein and L. Infeld, *The Evolution in Physics* (New York, 1938).

[5] The point has been repeatedly recognized by numerous scholars: e.g., Max Planck, *Scientific Autobiography and Other Papers* (New York, 1949), p. 152.

[6] Léon Walras, *Éléments d'économie politique pure* (3rd edn., Lausanne, 1896), p. 97. My translation.

[7] W. Stanley Jevons, *The Theory of Political Economy* (4th edn., London, 1924), p. 21 and p. 11.

[8] *Ibid.*, p. 21.

derelict if they had failed to try out "a fish hatchery in a flower bed." For trying, though not sufficient, is as absolutely necessary for the advancement of knowledge as it is for biological evolution. This is why we cannot cease to admire men like Jevons and Walras, or numerous others who even in physics hurried to adopt a new viewpoint without first testing their ground.[9] But our admiration for such unique feats does not justify persistence in a direction that trying has proved barren. Nor do we serve the interest of science by glossing over the impossibility of reducing the economic process to mechanical equations. In this respect, a significant symptom is the fact that Carl Menger is placed by almost every historian on a lower pedestal than either Walras or Jevons only because he was more conservative in treating the same problem, the subjective basis of value.[10] Moreover, in spite of the fact that no economy, not even that of a Robinson Crusoe, has been so far described by a Walrasian system in the same way in which the solar system has been described by a Lagrange system of mechanical equations, there are voices claiming that economics "has gone through its Newtonian revolution": only the other social sciences are still awaiting their Galileo or Pasteur.[11] Alfred North Whitehead's complaint that "the self-confidence of learned people is the comic tragedy of [our] civilization"[12] may be unsavory but does not seem entirely unfounded.

Opposition to Walras' and Jevons' claim that "economics, if it is to be a science at all, must be a mathematical science,"[13] has not failed to manifest itself. But, in my opinion, during the ensuing controversies swords have not been crossed over the crucial issue. For I believe that what social sciences, nay, all sciences need is not so much a new Galileo or a new Newton as a new Aristotle who would prescribe new rules for handling those notions that Logic cannot deal with.

This is not an extravagant vision. For no matter how much we may preen ourselves nowadays upon our latest scientific achievements, the evolution of human thought has not come to a stop. To think that we have even approached the end is either utter arrogance or mortifying pessimism. We cannot therefore write off the possibility of striking one

[9] Cf. P. W. Bridgman, *Reflections of a Physicist* (2nd edn., New York, 1955), p. 355.

[10] E.g., K. Wicksell, *Value, Capital and Rent* (London, 1954), p. 53; Joseph A. Schumpeter, *History of Economic Analysis* (New York, 1954), p. 918. Among the few exceptions: Frank H. Knight, "Marginal Utility Economics," *Encyclopaedia of the Social Sciences* (New York, 1931), V, 363; George J. Stigler, *Production and Distribution Theories* (New York, 1948), p. 134.

[11] Karl R. Popper, *The Poverty of Historicism* (Boston, 1957), p. 60 and note.

[12] Alfred North Whitehead, *Science and Philosophy* (New York, 1948), p. 103.

[13] Jevons, *Theory*, p. 3.

day upon the proper mutant idea that would lead to an anatomy of science capable of thriving equally well in natural as in social sciences. On rare occasions we find this hope more clearly expressed with the extremely pertinent remark that in such a unifying science physics will be "swallowed up" by biology, not the reverse.[14] Or, as Whitehead put it more sharply, "murder is a prerequisite for the absorption of biology into physics."[15] A historical precedent already exists: physicists and scientific philosophers had for a long time denied that "scientific" laws exist outside physics and chemistry, because only there did we find rigidly binding relations. Today they work hard to convince everybody that on the contrary the laws of nature are not rigid but stochastic and that the rigid law is only a limiting, hence highly special, case of the stochastic law. Somehow they usually fail to point out that the latter type of law is not a native of physical science but of the life sciences.

The history of human thought, therefore, teaches us that nothing can be extravagant in relation to what thought might discover or where. It is all the more necessary for us to recognize fully the source as well as the nature of our difficulty *at present*.

2. *Theoretical Science versus Science*. The first condition an environment must satisfy in order to sustain the life of a certain organism is to contain the chemical elements found in the anatomy of that organism. If it does not, we need not go any further. Let us, therefore, begin our inquiry by a "chemical" analysis of the anatomy of theoretical science.

As I have pointed out, the *causa materialis* of science, not only of theoretical science, consists of descriptive propositions. I have further explained that the distinctive feature of theoretical science is its logically ordered anatomy. Whoever is willing to look at the brute facts and accept some of their unpleasantness will agree that in some phenomenal domains an overwhelming majority of descriptive propositions do not possess the "chemical" properties required by logical ordering.

I can hardly overemphasize the fact that Logic, understood in its current Aristotelian sense, is capable of dealing only with one distinct class of propositions, such as

A. *The hypotenuse is greater than the leg,*

[14] Cf. J. S. Haldane, *The Sciences and Philosophy* (New York, 1929), p. 211. Also Erwin Schrödinger, *What Is Life?* (Cambridge, Eng., 1944), pp. 68 f; R. E. Peierls, *The Laws of Nature* (London, 1957), p. 277; L. von Bertalanffy, *Problems of Life* (New York, 1952), p. 153. Quite recently, G. P. Thomson, a Nobel laureate, ended his address at the Semicentennial lectures of Rice Institute (1962) by saying that "the future of physics lies with biology."

[15] Alfred North Whitehead, "Time, Space, and Material," in *Problems of Science and Philosophy*, Aristotelian Society, suppl. vol. 2, 1919, p. 45. See also Chapter V, Section 1, below.

but it is largely impotent when it comes to propositions such as

B. *Culturally determined wants are higher than biological wants,*

or

C. *Woodrow Wilson had a decisive influence upon the Versailles Peace Treaty.*

A logician would hardly deny this difference. But many, especially the logical positivists, would argue that propositions such as B or C are meaningless and, hence, the difference does not prove at all the limitation of Logic. This position is clearly explained by Max Black: *red* being a vague concept, the question "Is this color red?" has scarcely any meaning.[16] However, the use of the term "meaningless" for propositions that Logic cannot handle is a clever artifice for begging a vital question.

At bottom, the issue is whether knowledge is authentic only if it can be unified into a theory. In other words, is theoretical science the only form of scientific knowledge? The issue resolves into several questions: the first is what accounts for Logic's impotence to deal with "meaningless" propositions.

3. Numbers and Arithmomorphic Concepts. The boundaries of every science of fact are moving penumbras. Physics mingles with chemistry, chemistry with biology, economics with political science and sociology, and so on. There exists a physical chemistry, a biochemistry, and even a political economy in spite of our unwillingness to speak of it. Only the domain of Logic—conceived as *Principia Mathematica*—is limited by rigidly set and sharply drawn boundaries. The reason for this is that *discrete* distinction constitutes the very essence of Logic: perforce, discrete distinction must apply to Logic's own boundaries.

The elementary basis of *discrete* distinction is the distinction between two written symbols: between "m" and "n," "3" and "8," "excerpt" and "except," and so on. As these illustrations show, good symbolism requires perfect legibility of writing; otherwise we might not be able to distinguish without the shadow of a doubt between the members of the same pair. By the same token, spoken symbolism requires distinct pronunciation, without lisping or mumbling.

There is one and only one reason why we use symbols: to represent concepts visually or audibly so that these may be communicated from one

[16] Max Black, *The Nature of Mathematics* (New York, 1935), p. 100n. This position is frequently used to dodge basic questions, such as "Can a machine think?" See Chapter III, Section 10, below.

mind to another.[17] Whether in general reasoning or in Logic (i.e., formal logic), we deal with symbols qua representatives of *extant* concepts. Even in mathematics, where numbers and all other concepts are as distinct from one another as the symbols used to represent them, the position that numbers are nothing but "signs" has met with tremendous opposition.[18] Yet we do not go, it seems, so far as to realize (or to admit if we realize) that the fundamental principle upon which Logic rests is that *the property of discrete distinction should cover not only symbols but concepts as well.*

As long as this principle is regarded as normative no one could possibly quarrel over it. On the contrary, no one could deny the immense advantages derived from following the norm whenever possible. But it is often presented as a general law of thought. A more glaring example of Whitehead's "fallacy of misplaced concreteness" than such a position would be hard to find. To support it some have gone so far as to maintain that we can think but in words. If this were true, then thoughts would become a "symbol" of the words, a most fantastic reversal of the relationship between means and ends. Although the absurdity has been repeatedly exposed, it still survives under the skin of logical positivism.[19] Pareto did not first coin the word "ophelimity" and then think of the concept. Besides, thought is so fluid that even the weaker claim, namely, that we can coin a word for every thought, is absurd. "The Fallacy of the Perfect Dictionary"[20] is plain: even a perfect dictionary is molecular while thought is continuous in the most absolute sense. Plain also are the reason for and the meaning of the remark that "in symbols truth is darkened and veiled by the sensuous element."[21]

Since any particular real number constitutes the most elementary example of a discretely distinct concept, I propose to call any such concept *arithmomorphic*. Indeed, despite the use of the term "continuum" for the set of all real numbers, within the continuum every real number retains

[17] This limitation follows the usual line, which ignores tactile symbolism: taps on the shoulder, handshakes, etc. Braille and especially the case of Helen Keller prove that tactile symbolism can be as discretely distinct and as efficient as the other two. Its only shortcoming is the impossibility of transmission at a distance.

[18] Cf. the Introduction by P. E. B. Jourdain to Georg Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers* (New York, n.d.), pp. 20, 69 f; R. L. Wilder, *Introduction to the Foundations of Mathematics* (New York, 1956), ch. x and *passim*.

[19] For a discussion of the psychological evidence against the equation "thought = word," see Jacques Hadamard, *An Essay on the Psychology of Invention in the Mathematical Field* (Princeton, 1945), pp. 66 ff. For what it might be worth, as one who is multilingual I can vouch that I seldom think in any language, except just before expressing my thoughts orally or in writing.

[20] Alfred North Whitehead, *Modes of Thought* (New York, 1938), p. 235. See also P. W. Bridgman, *The Intelligent Individual and Society* (New York, 1938), pp. 69 f.

[21] G. W. F. Hegel, *Hegel's Science of Logic* (2 vols., London, 1951), I, 231.

a *distinct individuality* in all respects identical to that of an integer within the sequence of natural numbers. The number π, for instance, is discretely distinct from any other number, be it 3.141592653589793 or 10^{100}. So is the concept of "circle" from "10^{100}-gon" or from "square," and "electron" from "proton." In Logic "is" and "is not," "belongs" and "does not belong," "some" and "all," too, are *discretely* distinct.

Every arithmomorphic concept stands by itself in the same specific manner in which every "Ego" stands by itself perfectly conscious of its absolute differentiation from all other "Egos." This is, no doubt, the reason why our minds crave arithmomorphic concepts, which are as translucent as the feeling of one's own existence. Arithmomorphic concepts, to put it more directly, *do not overlap*. It is this peculiar (and restrictive) property of the material with which Logic can work that accounts for its tremendous efficiency: without this property we could neither compute, nor syllogize, nor construct a theoretical science. But, as happens with all powers, that of Logic too is limited by its own ground.

4. Dialectical Concepts. The antinomy between One and Many with which Plato, in particular, struggled is well known. One of its roots resides in the fact that the quality of discrete distinction does not necessarily pass from the arithmomorphic concept to its concrete denotations. There are, however, cases where the transfer operates. Four pencils are an "even number" of pencils; a concrete triangle is not a "square." Nor is there any great difficulty in deciding that Louis XIV constitutes a denotation of "king." But we can never be absolutely sure whether a concrete quadrangle is a "square."[22] In the world of ideas "square" is One, but in the world of the senses it is Many.[23]

On the other hand, if we are apt to debate endlessly whether a particular country is a "democracy" it is above all because the concept itself appears as Many, that is, it is not discretely distinct. If this is true, all the more the concrete cannot be One. A vast number of concepts belong to this very category; among them are the most vital concepts for human judgments, like "good," "justice," "likelihood," "want," etc. They have no arithmomorphic boundaries; instead, *they are surrounded by a penumbra within which they overlap with their opposites.*

At a particular historical moment a nation may be both a "democracy" and a "nondemocracy," just as there is an age when a man is both "young" and "old." Biologists have lately realized that even "life" has no arithmomorphic boundary: there are some crystal-viruses that con-

[22] Strangely, logicians do not argue that because of this fact "square" is a *vague* concept and "Is this quadrangle a square?" has no meaning.

[23] Plato, *Phaedrus*, 265D and, especially, *Republic*, VI. 507.

stitute a penumbra between living and dead matter.[24] Any particular want, as I have argued along well-trodden but abandoned trails, imperceptibly slides into other wants.[25] Careful thinkers do not hide that even in mathematics "the use of good judgment in determining when a statement form is acceptable in defining a class seems to be unavoidable."[26]

It goes without saying that to the category of concepts just illustrated we cannot apply the fundamental law of Logic, the Principle of Contradiction: "B cannot be both A and non-A." On the contrary, we must accept that, *in certain instances* at least, "B is both A and non-A" is the case. Since the latter principle is one cornerstone of Hegel's Dialectics, I propose to refer to the concepts that may violate the Principle of Contradiction as *dialectical*.[27]

In order to make it clear what we understand by dialectical concept, two points need special emphasis.

First, the impossibility mentioned earlier of deciding whether a concrete quadrangle is "square" has its roots in the imperfection of our senses and of their extensions, the measuring instruments. A *perfect* instrument would remove it. On the other hand, the difficulty of deciding whether a particular country is a democracy has nothing to do—as I shall explain in detail presently—with the imperfection of our sensory organs. It arises from another "imperfection," namely, that of our thought, which cannot always reduce an apprehended notion to an arithmomorphic concept. Of course, one may suggest that in this case too the difficulty would not exist for a *perfect* mind. However, the analogy does not seem to hold. For while the notion of a perfect measuring instrument is sufficiently clear (and moreover indispensable even for explaining the indeterminacy in physical measurements), the notion of a perfect mind is at most a verbal concoction. There is no direct bridge between an imperfect and the perfect measuring instrument. By the same token, the imperfect mind cannot know how a perfect mind would actually operate. It would itself become perfect the moment it knew how.

The second point is that a dialectical concept—in my sense—does not

[24] On the arithmomorphic definition of life, see Alfred J. Lotka, *Elements of Physical Biology* (Baltimore, 1925), chap. i and p. 218n.

[25] My essay entitled "Choice, Expectations and Measurability" (1954), reprinted in AE.

[26] L. M. Graves, *The Theory of Functions of Real Variables* (New York, 1946), p. 7. Also Henri Poincaré, *The Foundations of Science* (Lancaster, Pa., 1946), p. 479.

[27] The connection between dialectical concepts thus defined and Hegelian logic is not confined to this principle. However, even though the line followed by the present argument is inspired by Hegel's logic, it does not follow Hegel in all respects. We have been warned, and on good reasons, that one may ignore Hegel at tremendous risks. To follow Hegel only in part might very well be the greatest risk of all; yet I have no choice but to take this risk.

overlap with its opposite *throughout the entire range of denotations*. To wit, in most cases we can decide whether a thing, or a particular concept, represents a living organism or lifeless matter. If this were not so, then certainly dialectical concepts would be not only useless but also harmful. Though they are not *discretely distinct*, dialectical concepts are nevertheless *distinct*. The difference is this. A penumbra separates a dialectical concept from its opposite. In the case of an arithmomorphic concept the separation consists of a void: *tertium non datur*—there is no third case. The extremely important point is that the separating penumbra itself is a dialectical concept. Indeed, if the penumbra of A had arithmomorphic boundaries, then we could readily construct an arithmomorphic structure consisting of three discretely distinct notions: "proper A," "proper non-A," and "indifferent A." The procedure is most familiar to the student of consumer's choice where we take it for granted that between "preference" and "nonpreference" there *must* be "indifference."[28]

Undoubtedly, a penumbra surrounded by another penumbra confronts us with an infinite regress. But there is no point in condemning dialectical concepts because of this aspect: in the end the dialectical infinite regress resolves itself just as the infinite regress of Achilles running after the tortoise comes to an actual end. As Schumpeter rightly protested, "there is no sense in our case in asking: 'Where does that type [of entrepreneur] begin then?' and then to exclaim: 'This is no type at all!'"[29] Should we also refuse to recognize and study virtue and vice just because there is no sharp division—as Hume, among many, observed[30]—between these two opposing qualities of the human spirit? Far from being a deadly sin, the infinite regress of the dialectical penumbra constitutes the salient merit of the dialectical concepts: as we shall see, it reflects the most essential aspect of Change.

5. *Platonic Traditions in Modern Thought*. To solve the perplexing problem of One and Many, Plato taught that ideas live in a world of their own, "the upper-world," where each retains "a permanent individuality" and, moreover, remains "the same and unchanging."[31] Things of the "lower-world" partake of these ideas, that is, resemble them.[32] The pivot

[28] Cf. my essay "Choice, Expectations and Measurability" (1954), reprinted in AE.

[29] Joseph A. Schumpeter, *The Theory of Economic Development* (Cambridge, Mass., 1949), p. 82n.

[30] David Hume, *Writings on Economics*, ed., E. Rotwein (London, 1955), p. 19.

[31] *Phaedo*, 78, *Philebus*, 15. Plato's doctrine that ideas are "fixed patterns" permeates all his Dialogues. For just a few additional references, *Parmenides*, 129 ff, *Cratylus*, 439–440.

[32] *Phaedo*, 100 ff. It is significant that although Plato (*Phaedo*, 104) illustrates the discrete distinction of ideas by referring to integral numbers, he never discusses the problem why some things partake fully and others only partly of ideas.

of Plato's epistemology is that we are born with a latent knowledge of all ideas—as Kant was to argue later about some notions—because our immortal soul has visited their world some time in the past. Every one of us, therefore, can learn ideas by reminiscence.[33]

Plato's extreme idealism can hardly stir open applause nowadays. Yet his mystical explanation of how ideas are revealed to us in their purest form underlies many modern thoughts on "clear thinking." The Platonic tenet that only a privileged few are acquainted with ideas but cannot describe them publicly, is manifest, for example, in Darwin's position that "species" is that form which is so classified by "the opinion of naturalists having sound judgment and wide experience."[34] Even more Platonic in essence is the frequently heard view that "constitutional law" has one and only one definition: it is the law pronounced as such by the U.S. Supreme Court if and when in a case brought before it the Court is explicitly asked for a ruling on this point.

There can be no doubt about the *fact* that a consummate naturalist or a Supreme Court justice is far more qualified than the average individual for dealing with the problem of species or constitutional law. But that is not what the upholders of this sort of definition usually mean: they claim that the definitions are operational and, hence, dispose of the greatest enemy of clear thought—vagueness. It is obvious, however, that the claim is specious: the result of the defining operation is not One but Many.[35] Not only is the operation extremely cumbersome, even wholly impractical at times, but the definition offers no enlightenment to the student. Before anyone becomes an authority on evolution, and even thereafter, he needs to know what "fitness" means without waiting until natural selection will have eliminated the unfit. Science cannot be satisfied with the idea that the only way to find out whether a mushroom is poisonous is to eat it.

Sociology and political science, in particular, abound in examples of another form of disguised Platonic rationale. For instance, arguments often proceed, however unawares, from the position that the pure idea of "democracy" is represented by one particular country—usually the writer's: all other countries only partake of this idea in varying degrees.

Plato's *Dialogues* leave no doubt that he was perfectly aware of the fact that we know concepts either by definition or by intuition. He realized that since definition constitutes a public description, anyone may

[33] *Meno*, 81–82, *Phaedo*, 73 ff, *Phaedrus*, 249–250.

[34] Charles Darwin, *The Origin of Species* (6th edn., London, 1898), p. 34.

[35] As Charles Darwin himself observes in a different place, *The Descent of Man* (2nd edn., New York, n.d.), p. 190: Thirteen eminent naturalists differed so widely as to divide the human species into as few as two and as many as sixty-three races!

learn to know a concept by definition. He also realized that we can get acquainted with some concepts only by direct apprehension supplemented by Socratic analysis.[36] Plato's difficulty comes from his belief *that regardless of their formation all concepts are arithmomorphic*, that "everything resembles a number," as his good friend Xenocrates was to teach later. One Dialogue after another proves that although Plato was bothered by the difficulties of definition in the case of many concepts, he never doubted that in the end all concepts can be defined. Very likely, Plato—like many after him—indiscriminately extrapolated the past: since all defined concepts have at one time been concepts by intuition, all present concepts by intuition must necessarily become concepts by definition.

The issue may be illustrated by one of our previous examples. Should we strive for an arithmomorphic concept of "democracy," we would soon discover that no democratic country fits the concept: not Switzerland, because Swiss women have no voting right; not the United States, because it has no popular referendum; not the United Kingdom, because the Parliament cannot meet without the solemn approval of the King, and so on down the line. The penumbra that separates "democracy" from "autocracy" is indeed very wide. As a result, "even the dictatorship of Hitler in National-Socialist Germany had democratic features, and in the democracy of the United States we find certain dictatorial elements."[37] But this does not mean that Hitlerite Germany and the United States must be thrown together in the same conceptual pot, any more than the existence of a penumbra of viruses renders the distinction between "man" and "stone" senseless.

Furthermore, the efforts to define democracy are thwarted by a more general and more convincing kind of difficulty than that just mentioned. Since "democracy" undoubtedly implies the right to vote but not for all ages, its definition must necessarily specify the *proper* limit of the voting age. Let us assume that we agree upon L being this limit. The natural question of why $L-\epsilon$ is not as good a limit fully reveals the impossibility of taking care of all the imponderables of "democracy" by an arithmomorphic concept.

Of "democracy" as well as of "good," "want," etc., we can say what St. Augustine in essence said of Time: if you know nothing about it I cannot tell you what it is, but if you know even vaguely what it means let us talk about it.[38]

[36] *Republic*, VI. 511. In all probability, it was this sort of analysis that Plato meant by "dialectics," but he never clarified this term.

[37] Max Rheinstein in the "Introduction" to Max Weber, *On Law in Economy and Society* (Cambridge, Mass., 1954), p. xxxvi.

[38] Saint Augustine, *Confessions*, XI. 17.

6. *Dialectical Concepts and Science.* No philosophical school, I think, would nowadays deny the existence of dialectical concepts as they have been defined above. But opinions as to their relationship to science and to knowledge in general vary between two extremes.

At one end we find every form of positivism proclaiming that whatever the purpose and uses of dialectical concepts, these concepts are antagonistic to science: knowledge proper exists only to the extent to which it is expressed in arithmomorphic concepts. The position recalls that of the Catholic Church: holy thought can be expressed only in Latin.

At the other end there are the Hegelians of all strains maintaining that knowledge is attained only with the aid of dialectical notions in the strict Hegelian sense, i.e., notions to which the principle "A is non-A" applies *always*.

There is, though, some definite asymmetry between the two opposing schools: no Hegelian—Hegel included—has ever denied either the unique ease with which thought handles arithmomorphic concepts or their tremendous usefulness.[39] For these concepts possess a built-in device against most kinds of errors of thought that dialectical concepts do not have. Because of this difference we are apt to associate dialectical concepts with loose thinking, even if we do not profess logical positivism. The by now famous expression "the muddled waters of Hegelian dialectics" speaks for itself. Moreover, the use of the antidialectical weapon has come to be the easiest way for disposing of someone else's argument.[40] Yet the highly significant fact is that no one has been able to present an argument against dialectical concepts *without incessant recourse to them!*

We are badly mistaken if we believe that the presence of such terms as "only if" or "nothing but" in a sentence clears it of all "dialectical nonsense." As an eloquent example, we may take the sentence "A proposition has meaning only if it is verifiable," and the sentence "When we speak of verifiability we mean *logical* possibility of verification, and nothing but this,"[41] which together form the creed of the Vienna posi-

[39] That Hegel's philosophy has been made responsible for almost every ideological abuse and variously denounced as "pure nonsense [that] had previously been known only in madhouses" or as "a monument to German stupidity" need not concern us. (Will Durant, in *The Story of Philosophy*, New York, 1953, p. 221, gives E. Caird, *Hegel*, London, 1883, as the source of these opinions; all my efforts to locate the quotation have been in vain.) But I must point out that the often-heard accusation that Hegel denied the great usefulness of mathematics or theoretical science is absolutely baseless: see *The Logic of Hegel*, tr. W. Wallace (2nd edn., London, 1904), p. 187.

[40] Precisely because I wish to show that the sin is not confined to the rank and file, I shall mention that Knight within a single article denounces the concept of instinct as arbitrary and unscientific but uses the concept of want freely. Frank H. Knight, *The Ethics of Competition* (New York, 1935), p. 29 and *passim*.

[41] Moritz Schlick, "Meaning and Verification," *Philosophical Review*, XLV (1936), 344, 349.

tivism. If one is not a positivist, perhaps he would admit that there is some sense in these tenets, despite the criticism he may have to offer. But if one is a full-fledged positivist, he must also claim that "the dividing line between logical possibility and impossibility of verification is *absolutely sharp and distinct*; there is no gradual transition between meaning and nonsense."[42] Hence, for the two previous propositions to have a meaning, we need to describe "the logical possibility of [their] verification" in an absolutely sharp and distinct manner. To my knowledge, no one has yet offered such a description. Positivism does not seem to realize at all that the concept of verifiability—or that the position that "the meaning of a proposition is the method of its verification"[43]—is covered by a dialectical penumbra in spite of the apparent rigor of the sentences used in the argument. Of course, one can easily give examples of pure nonsense—"my friend died the day after tomorrow" is used by Moritz Schlick—or of pure arithmomorphic sense. However—as I have argued earlier—this does not dispose of a dialectical penumbra of graded differences of clearness between the two extreme cases. I hope the reader will not take offense at the unavoidable conclusion that most of the time all of us talk some nonsense, that is, express our thoughts in dialectical terms with no clear-cut meaning.

Some of the books written by the very writers who—like Bertrand Russell or Bridgman, for example—have looked upon combatting vagueness in science as a point of highest intellectual honor, constitute the most convincing proof that correct reasoning with dialectical concepts is not impossible.[44] In connection with this thesis of mine and in relation to the positivist viewpoint mentioned earlier (Section 2) that "this color is red" is a meaningless proposition, let me refer the reader to one of the most appreciated articles of Bertrand Russell: "Not only are we aware of particular yellows, but if we have seen a sufficient number of yellows and have sufficient intelligence, we are aware of the universal *yellow*; this universal is the subject in such judgments as 'yellow differs from blue' or 'yellow resembles blue less than green does.' And the universal yellow is the predicate in such judgments as 'this is yellow.'"[45] Although a positivist would certainly make no sense of this chain of dialectical concepts, this is what I would call dialectical reasoning at its best (with the risk of making Mr. Bertrand Russell feel offended thereby). And the important

[42] *Ibid.*, 352. My italics.

[43] *Ibid.*, 341.

[44] E.g., Bertrand Russell, *Principles of Social Reconstruction* (London, 1916), and P. W. Bridgman, *The Intelligent Individual and Society*.

[45] Bertrand Russell, *Mysticism and Logic* (New York, 1929), p. 212. [Concerning the next remark in the text, the reader should know that Bertrand Russell was alive when this volume went to press.]

fact is that such a reasoning is a far more delicate operation than syllogizing with arithmomorphic concepts. As I shall argue later on (Chapter III, Section 10), it constitutes the most important quality that differentiates the human mind from any mechanical brain.

Long ago, Blaise Pascal pointed out the difference between these two types of reasoning as well as their correlation with two distinct qualities of our intellect: *l'esprit géométrique* and *l'esprit de finesse*.[46] To blame dialectical concepts for any muddled thinking is, therefore, tantamount to blaming the artist's colors for what the artless—and even the talented at times—might do with them. As to the artful use of dialectical concepts by sophists of all strains, we have been long since instructed by Socrates on the difference between "the mere art of disputation and true dialectics."[47]

Now, both *l'esprit géométrique* and *l'esprit de finesse* are acquired (or developed) through proper training and exposure to as large a sample of ideas as possible. And we cannot possibly deny that social scientists generally possess enough *esprit de finesse* to interpret correctly the proposition "democracy allows for an equitable satisfaction of individual wants" and to reason correctly with similar propositions where almost every term is a dialectical concept. (And if some social scientists do not possess enough *esprit de finesse* for the job, God help them!) The feat is not by any means extraordinary. As Bridgman once observed, "little Johnnie and I myself know perfectly well what I want when I tell him to be good, although neither of us could describe exactly what we meant under cross-examination."[48]

The position that dialectical concepts should be barred from science because they would infest it with muddled thinking is, therefore, a flight of fancy—unfortunately, not an innocuous one. For it has bred another kind of muddle that now plagues large sectors of social sciences: arithmomania. To cite a few cases from economics alone. The complex notion of economic development has been reduced to a number, the income per capita. The dialectical spectrum of human wants (perhaps the most important element of the economic process) has long since been covered under the colorless numerical concept of "utility" for which, moreover, nobody has yet been able to provide an actual procedure of measurement.

7. *Probability: An Illustration of Hegelian Dialectics.* Nothing could illustrate the argument of the foregoing section more sharply than the concept of probability, now common to all special sciences. There are, as we all know, a multiplicity of largely antagonistic "doctrines," each claiming

[46] *Pensées*, 1–2, in Blaise Pascal, *Oeuvres complètes*, ed. J. Chevalier (Paris, 1954), pp. 1091 ff.

[47] Plato, *Philebus*, 17; more on this in *Theaetus*, 167–168.

[48] Bridgman, *Intelligent Individual and Society*, p. 72; also pp. 56 ff.

that only its own approach leads to what probability means (or, rather, should mean). The claim should not surprise us. Without going into lengthy details here, let me observe that, antagonistic though these doctrines are, they all have in fact the same objective: to order expectations with the aid of some numerical coefficient which each doctrine calls "probability." Expectation, however, is a complex state of the human mind involving two distinct elements: E, that part of the individual's knowledge of which he is aware at the time of the expectation, and P, an assertive proposition about a fact or an event usually, but not necessarily, uncertain. Symbolically, the expectation of an individual, I, may then be represented by $\mathscr{E}(I, E, P)$.[49]

In one group of doctrines—the Personalistic and the Subjectivistic, as they are called—attention is focused on I and probability is defined as the "degree of belief" the individual has in the fact or the event asserted by P. Another category leaves out I as well as that part of E that is not language and defines probability as a measure of the "truth" expressed by P, in fact, as a coefficient computed according to some (largely arbitrary) syntactical recipe. Needless to add, none of these doctrines is free from assumptions that fly in the face of elementary facts; some amount to little more than an exercise, however delightful, in empty axiomatization.

The only doctrines that should retain our attention here are those which may be called "Objectivistic" because they define probability independently of I (and of the kind of language used by I). In my formalization of expectation, the objective coefficient of probability—whenever it can be determined according to the rules provided and is also known by I—is part of E. The important point is that the ordering of the individual's expectations is a consequence of the arithmetical ordering of probabilities, not vice versa (as is the case in the Subjectivistic or Personalistic doctrines).[50]

The contest in the objectivistic approach is between the Laplacean and the Frequentist doctrines. The main criticism against the Laplacean definition of probability as the ratio between the number of favorable cases and that of all cases, "all cases being equally probable," cannot be refuted on Logical grounds: the definition *is* circular. The criticism, I contend, is idle because objective probability is basically a dialectical notion in the Hegelian sense. Indeed, the Frequentist definition, too, is circular if formulated properly.

[49] See my article "The Nature of Expectation and Uncertainty" (1958), reprinted in AE, where I presented a general critique of the main doctrines of probabilities. In addition to the present section, Chapter VI and Appendix F in the present volume contain some further thoughts on this topic.

[50] More on this in Appendix F.

In the Frequentist doctrine the probability of an event is defined by the *mathematical* relation

$$(D_1) \qquad p = \lim f_n \quad \text{for } n \to \infty,$$

where f_n is the relative frequency of the event in the first n observations of an infinite sequence of observations under invariable conditions.[51] Although the domain of application of probability is thereby greatly restricted, the doctrine has a double merit—it relates the concept directly to observed facts and to a number. For this reason, it won an overwhelming acceptance from the outset and was embraced wholeheartedly by all statisticians. With modern physics taking the position that phenomena at the level of the elementary particles are governed only by probabilistic laws which reflect an irreducible random factor in nature, *not our ignorance of some hidden variables*,[52] the Frequentist doctrine set a claim to epistemological completeness.

"Probabilities are as real as masses," said H. Margenau.[53] Yet the truth is rather the reverse: masses are as real as probabilities. Indeed, anything we can now say about masses depends on what we can say about probabilities. "The mass of the mu meson is 200 times that of the electron," for instance, is a proposition that involves the probability of an observation showing that the mass of a mu meson is, say, 195 times that of an electron. The verification of propositions about probabilities is, therefore, the only fundamental issue. Everything else depends upon this verification.

[51] Obviously, the statement does not imply that *all* expectations are thereby ordered; for some P's the probabilities may not exist according to the rules provided or may not be part of E.

[52] Uncertainty in quantum physics "is not due to our human ignorance: it is *objectively uncertain* when [a particular] atom will disintegrate." F. Waismann, "The Decline and Fall of Causality," *Turning Points in Physics*, ed. R. J. Blin-Stoyle, *et al.* (Amsterdam, 1959), p. 141. A theorem proved in 1932 by J. von Neumann (*Mathematical Foundations of Quantum Mechanics*, Princeton, 1955, pp. 323–325), according to which the present laws of quantum mechanics are incompatible with the thought that they may hide some causal variables, fostered the belief that these laws represent a definitive limit to a causal explanation of the behavior of elementary matter (cf. Louis de Broglie, *Physics and Microphysics*, London, 1955, pp. 195–205). However, because it implies that there can be no breakthrough at the subquantum level, Neumann's "dead-end" theorem should have been suspect from the outset. Actually, Broglie—who first saluted it enthusiastically—found a loophole in the whole argument (Broglie, *New Perspectives in Physics*, New York, 1962, pp. 99–102). For a more comprehensive discussion of the point, see David Bohm, *Causality and Chance in Modern Physics* (London, 1957), pp. 95–97.

The opposite idea, that chance reflects only our inability to solve the inextricable system of equations that govern phenomena or to know all the factors involved, is associated with the name of Henri Poincaré (*The Foundations of Science*, pp. 159 f, 395 f).

[53] H. Margenau, *Open Vistas: Philosophical Perspectives of Modern Science* (New Haven, Conn., 1961), p. 183n.

Recalling a point made earlier, let us ask a positivist which method of verification we should use, according to his philosophy, for the familiar proposition "the probability of this coin to show heads is 1/2." He could not possibly answer that we should perform an infinite number of observations, as required by the definition (D_1), for in that case he would implicitly admit that there are propositions that cannot be verified and, hence, cannot be classified as either sense or nonsense. Nor could he tell us to perform "a *sufficiently large number* of observations," because, among other things, he would be caught flagrante delicto of using a dialectical concept! But let us see what the typical answer tells us in essence: "If a coin were thrown a thousand times and the head came up 490 times, we would regard this as supporting the hypothesis that the probability of its coming up is 1/2 . . .; but if it came up only 400 times we would normally reject the hypothesis . . . We proceed in this manner because . . . there is a tacit acceptance of *some* degree of allowable deviation, however *vaguely* we may formulate it to ourselves."[54] The dialectical cat is thus let out of the positivist bag.

To be sure, the calculus of probabilities provides us with a number which statisticians call degree of confidence in a hypothesis. But what particular degree of confidence draws "the absolutely sharp and distinct line" between a verified and a false proposition is a question that has to be answered before positivists can make good their claim about the absolute distinction between "meaning and nonsense." This is not all, however. From the fundamental theorems of probability calculus—which are endorsed by the Frequentist doctrine, too—it follows that a coin that has been verified to be fair with an immensely great degree of confidence might nevertheless show only "heads" for an infinite number of times.[55] The Frequentist definition, therefore, harbors a contradiction of Logic. And this contradiction will stay with us as long as we refuse to recognize that since any proposition on probability—understood as a physical coordinate—turns to probability for its verification, the definition of probability has to be circular.

The basic fault of the Frequentist doctrine resides in not seeing that, *viewed as a whole, a sequence of observations is a random event just as a single observation is*. And just as a single observation may be erratic, so may a sequence be. Therefore, we must allow for the fact that in some cases f_n may have a limit different from p or no limit at all. But we need to add that the probability of a sequence for which f_n tends toward p is very

[54] Ernest Nagel, "The Meaning of Probability," *Journal of American Statistical Association*, XXXI (1936), 22. "Vaguely" italicized by me.

[55] For the apparent contradiction between this statement and the fact that the probability of such an occurrence is "zero," see Appendix A, para. 1 and 13.

large, and the longer the sequence is, the larger is this probability. More exactly, this probability tends to unity as n tends toward infinity. Let us therefore posit the following:

If E is a random event, there exists a number p such that for any positive numbers ϵ *and* δ *there is an integer N such that*

$$(D_2) \qquad 1 > \text{Prob}\,[|f_n - p| < \epsilon] > 1 - \delta$$

for any $n > N$.

A few things should be made clear about this proposition. First, the condition that the middle term in (D_2) should be smaller than unity is indispensable. Only in this way can we acknowledge that erratic sequences must occur at times. Second, the proposition must be viewed as a law of nature for which we may reserve the name of the Law of Large Numbers and, thus, put an end to the confusion ordinarily surrounding the nature of this law. Third, we may regard the proposition as a definition of physical probability. We need only observe that this definition includes both the beginning and the end of a complete thought, in order to see that there is no way of conceiving probability other than in the purest Hegelian sense. If probability is the ultimate element of nature, then forcibly its definition must rest on probability.

And if the centuries-old struggle with the problem of finding an *analytical* definition of probability has produced only endless controversies between the various doctrines, it is, in my opinion, because too little attention has been paid to the singular notion of random. For the dialectical root, in fact, lies in this notion: probability is only an arithmetical aspect of it.

That the notion of random involves an irreducible contradiction is beyond question. To begin with, random order must be so irregular as to exclude any possibility of representing it by an analytical formula. This is the essence of the highly interesting observation of Borel that the human mind is incapable of imitating the hazard.[56] But long ago Joseph Bertrand queried, "How dare we speak of the laws of hazard? Is not hazard the antithesis of any law whatsoever?"[57] The answer to Bertrand's query is that random does not mean wild haphazard, that is, complete absence of order. The opposition between the thesis of random's irregularity and the antithesis of random's peculiar order finds its synthesis in the notion of

[56] Émile Borel, "Sur l'imitation du hasard," *Comptes Rendus, Académie des Sciences*, CCIV (1937), 203–205. However, Borel's claim (Émile Borel, "Les probabilités" *Encyclopédie Française*, I, 1. 96-4) that he has offered a demonstration of that impossibility is spurious.

[57] Joseph Bertrand, *Calcul des probabilités* (Paris, 1889), p. vi. My translation.

probability. From this comes the circularity of the definition of probability, whether in the Laplacean or the Frequentist form.

Another upshot of the preceding argument is that the opposition between Poincaré's view—that random is a residual of the imperfection or incompleteness of our knowledge—and the tenet of modern physics—that random is an intrinsic aspect of the mode of being of things in general—is fictitious. As was often the case with him, Poincaré did offer some startling demonstrations of how statistical permanences may arise from causal relationships if very small variations in the initial conditions of the system (say, a roulette wheel) produce appreciably different outcomes.[58] All these proofs assume, however, that the initial conditions are subject to some, not necessarily known, probabilistic law. That is, Poincaré did not create random from a purely causal structure. And in rounding up the justification of his position that random is connected with ignorance, Poincaré explained that the information supplied by probabilities "will not cease to be true upon the day when these [fortuitous] phenomena shall be better known."[59] The statement lays bare the crux of the whole matter. For let us suppose that one day we discover some subquantum phenomena that will enable us to predict which atom(s) of radium shall disintegrate next. We shall still have to explain why the disintegration of atoms left to themselves follows a random order.

We can understand then why, ever since the dawn of statistical mechanics, physicists have shown a marked weakness for the idea that random can be generated by a system governed by causal relationships *alone*. Among the numerous attempts to justify this idea, that of David Bohm provides a good instructive ground. In explaining that if automobile accidents are unpredictable it is only because we can neither ascertain nor take account *ex ante* of all the numerous factors that *ex post* explain each individual accident, Bohm merely follows Poincaré.[60] The same is true of his position that the world is governed by infinitely many laws and, hence, there is always an infinity of laws or factors that remain beyond the reach of science.[61] What he tries to build on this foundation, however, is not always clear. He says that "the assumption that the laws of nature constitute an infinite series of smaller and smaller steps that approach what is in essence a mechanistic limit is just as arbitrary and unprovable as is the assumption of a finite set of laws permitting an exhaustive

[58] Henri Poincaré, *Calcul des probabilités* (Paris, 1912), pp. 146–152, and Poincaré, *Foundations of Science*, pp. 403–406.

[59] Poincaré, *Foundations of Science*, p. 396.

[60] Bohm, *Causality and Chance*, pp. 2 f, 21 ff. See also D. Bohm and W. Schützer, "The General Statistical Problem in Physics and the Theory of Probability," *Nuovo Cimento*, Suppl. Series X, II (1955), 1006–1008.

[61] Bohm, *Causality and Chance, passim*.

treatment of the whole of nature."[62] This statement proves that Bohm implicitly recognizes the existence of an irreducible random *residual*. But then his subsequent statement that randomness is the result of the fact that the infinitely many factors "left out of any such system of [finite] theory are in general undergoing some kind of a random fluctuation,"[63] is puzzling. For if we assume a random residual, two conclusions follow immediately: first, the additional assumption of infinitely many laws governing the same phenomenon is no longer necessary to explain random, and second, regardless of how many factors are left out of account the deviations of the observed from the "theoretical" values are not purely random errors. The reason why Bohm brings in the infinity of laws is that he wants to justify the last proposition on the "well-known theorem [according to which] the effects of chance fluctuations tend to cancel out."[64]. However, the famous theorem has power over actuality if and only if each effect is produced by a *random* cause, which must be defined independently of the theorem. And if each cause is subject to random, again we do not need an infinity of them for explaining random. Like many other writers on probability, Bohm seems to confuse here an abstract mathematical theorem with the actual behavior of nature. This confusion is neatly obvious in his claim (so dear to many physicists) that a "determinate law" always generates random provided that the mechanism governed by it is such that extremely small variations in the initial conditions produce appreciably different results. As is the case in similar arguments by others, what he proves in fact is an ergodic geometrical theorem which, needless to say, is in antithetical opposition to the idea of random.[65]

Curiously, the authors who set out to prove the reducibility of random to causality usually raise a corner of the veil that covers the fallacy of their formal arguments. Thus Bohm seems to ignore that mechanics alone cannot justify the proposition that, because of the symmetry of the die

[62] *Ibid.*, p. 134. In this connection, we may recall the opposite belief shared by many statisticians in the applied fields who, explicitly or implicitly, think that if the regression function would include all "nonspecified factors" a perfect correlation (i.e., a totally determined relationship) would obtain. The thought is that the product

$$1 - R^2_{1.23\ldots n} = (1 - r^2_{12})(1 - r^2_{13.2})\cdots(1 - r^2_{1n.23\ldots n-1}),$$

where R and the r's are the standard notations for correlation coefficients, must tend toward zero for $n \to \infty$ since it decreases with every successive factor, $1 - r^2_{1n.23\ldots n-1} < 1$. However, on purely mathematical grounds, the limit of that product need not be zero.

[63] *Ibid.*, p. 141; also Bohm and Schützer, p. 1008

[64] Bohm, p. 23.

[65] Bohm and Schützer, pp. 1024 ff. More on this issue in Chapter VI, Section 3, below.

and the complexity of the hand's motion, "in the long run and in the average, these fluctuations [of the outcome] favor no particular face."[66] On the other hand, to take this proposition as an independent basis for random and probability is to go back to old man Laplace and his subjective Principle of Insufficient Reason.

The only way out is to face the fact that in the mode of being of the actual world there is an order which, because of its dialectical nature in the Hegelian sense, cannot be represented by an analytical (strictly causal) formula. Like the pain which cannot exist either without the needle or without the sentient being, random is a relational element. The two opposing views on random, about which we have spoken here, are the two ends of one and the same bridge between human understanding and the actual world.

[66] Bohm and Schützer, p. 1011. The most convincing counter example of this thesis is the fact that even though the sequence of Monte Carlo "random" numbers is constructed by a procedure that satisfies both the condition of instability and of statistical trends, the sequence in the end hits a constant run. The situation is entirely analogous to Poincaré's famous slip in asserting that the third decimal digits in a logarithm table form a random sequence (Poincaré, *Foundations of Science*, pp. 161 f).